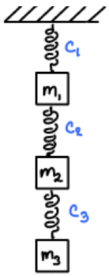
Spring Systems - Elongation and Equilibrium

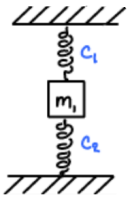
1 ELONGATION AND STIFFNESS MATRICES.

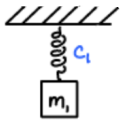
For the spring systems below (1) write the elongation matrix, (2) write the stiffness matrix, and (3) draw the associated weighted directed graph.
Assume that the spring constants are $c_k = k$ (i.e. $c_1 = 1$, $c_2 = 2$, etc). Let down be the positive direction.

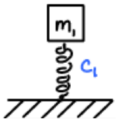
(a) 

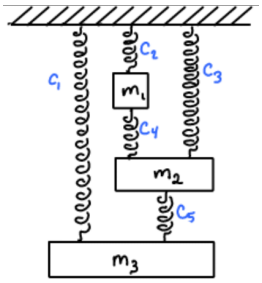
(b) 

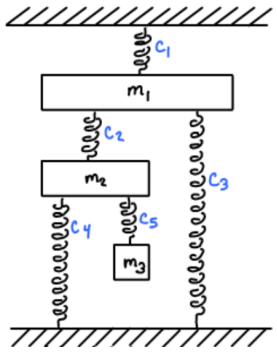
(c) 

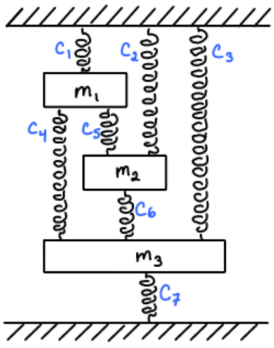
(d) 

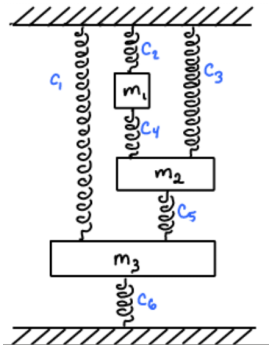
(e) 

(f) 

(g) 

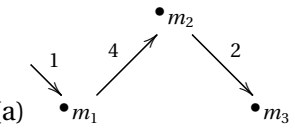
(h) 

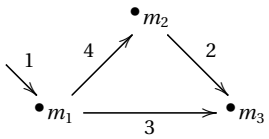
(i) 

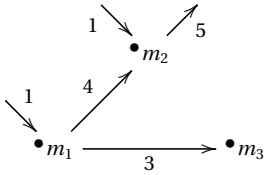
(j) 

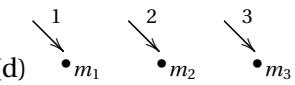
2 SPRING SYSTEMS FOR GRAPHS

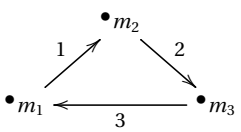
Draw spring systems for the weighted directed graphs below.

(a) 

(b) 

(c) 

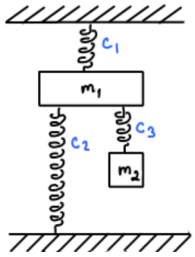
(d) 

(e) 

(This one is tricky...)

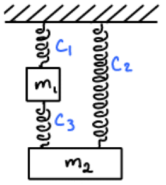
3 Force Caused by Displacement.

Calculate the forces on each mass required for the displacements given below. (Only vertical motion is allowed.)

(a) 

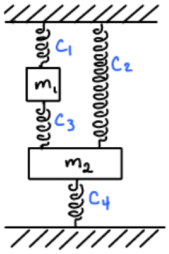
$c_1 = 2$, $c_2 = 1$, $c_3 = 1$

$u_1 = -1$, $u_2 = 2$

(b) 

$c_1 = 2$, $c_2 = 3$, $c_3 = 1$

$u_1 = 2$, $u_2 = -1$

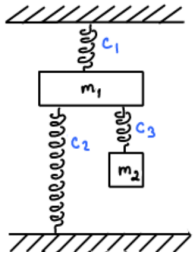
(c) 

$c_1 = 2$, $c_2 = 1$, $c_3 = 3$, $c_4 = 1$

$u_1 = 1$, $u_2 = -1$

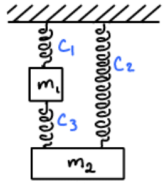
4 Displacement Caused by Force.

Calculate the displacement of each mass resulting from the forces given below. Also compute the resulting elongations of the springs. (Only vertical motion is allowed.)

(a) 

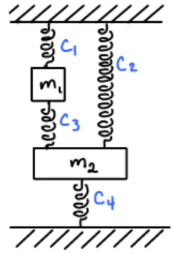
$c_1 = 2$, $c_2 = 1$, $c_3 = 1$

$f_1 = 5$, $f_2 = 1$

(b) 

$c_1 = 2$, $c_2 = 3$, $c_3 = 1$

$f_1 = 7$, $f_2 = 5$

(c) 

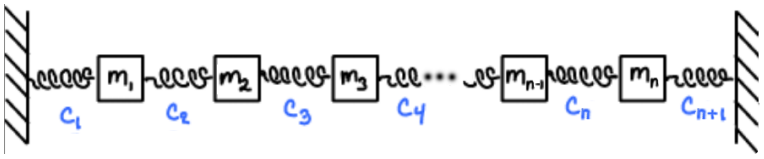
$c_1 = 2$, $c_2 = 1$, $c_3 = 3$, $c_4 = 1$

$f_1 = -8$, $f_2 = 8$

5 Long Line of Masses

This is an extra problem for **expert** students which makes a connection between the spring systems we are solving now and the discrete differential equations which we will solve later in the course.

Consider a very long line of equal masses and springs as drawn below (let $c_1 = \cdots = c_{n+1} = 1$).



For the questions below it may help to first consider the cases where $n = 2$ and $n = 3$.

- Write the elongation matrix and stiffness matrix.
- What force is required to displace only one mass ($u_k = 1$ and all other $u_i = 0$)?
- What is the displacement caused by a force at only one mass ($f_k = 1$ and all other $f_i = 0$)?
- How do the answers above change if we remove the last spring c_{n+1}?
- What if we remove the first **and** last springs?

You will see these matrices again when we discuss discretization of differential equations!

Explanation: Displacement of a line of masses is given by the discrete function $f(i) = u_i$. Solving many basic 2nd order differential equations can be viewed calculating the displacement caused by forces on a long line of masses where the springs have certain strengths!